MULTIDISCIPLINARY JOURNAL OF ENGINEERING SCIENCES

ARTICLE
Vol. 1 (2023), 36-46
DOI:10.5281/zenodo.7803023

Numerical investigation of unsteady flow and heat transfer of a free convective second-grade fluid passing through exponentially accelerated vertical porous plate

Ashirbad Kumar Rath [1]*, Itishree Nayak [1], Sukanya Padhi[1]

[1] *Veer Surendra Sai University of Technology, Burla 768018, Odisha, India*

**** Corresponding author: akrath_phdmath@vssut.ac.in***

Received: 4 February 2023
Revised: 28 February 2023
Accepted: 30 March 2023
Published online: 30 March 2023
Published in issue: 31 March 2023

Abstract

The constitutive equation considered here exhibits the heat transfer and unsteady flow of a second-grade fluid past through a long porous wall vertically. The discretized form of equations is obtained by implementing the method of finite difference of Crank -Nicolson type and solved numerically by resorting highly convergent method called “damped Newton". Two different cases are taken into consideration i.e the plate is accelerating differently at (n = 0.5) and constant acceleration (n = 1) and a comparative study is performed for the obtained results. Influence of various parameters g_r, R, P_r and α on the temperature and velocity field are studied through several graphs. The significant finding of the study is that for large P_r values, increases in the viscoelastic parameter α cause a rise in the velocity, Still, a contradicting effect is observed for comparatively smaller P_r values.

Keywords: second-grade fluids, free convective flow, exponentially fitted scheme, damped-Newton method, finite difference method

1. Introduction

In the present scenario, study of non Newtonian fluid rheology is intensified owing to its numerous applications in industrial mechanisms, chemical engineering and medical sciences. It is not possible to analyse all the diversified complex properties of non Newtonian fluid through a single model and therefore many researchers have proposed different fluid models. The second-grade fluid is known as one of the simplest visco-elastic fluid structure exhibiting peculiar properties that fascinate numerous researchers for its importance in the technological application and composite mathematical features [18, 21]. Beavers and Joseph [6] established a practical method of viscometry to study the characteristics of non Newtonian fluids. Teipel [20] obtained the stagnation point of the second-order fluid and resolved using a similarity approach. Bandelli and Rajagopal [5] analysed various unidirectional second-grade fluids in the finite dimension domain with the help of different integral transformations. They derived the Existence and Uniqueness of second-grade fluid flow for slip boundary conditions. Mohapatra et al. [15] obtained a solution numerically for MHD free convective flow of second-grade fluid passing over an infinite vertical plate. Massoudi and Maneschy [13] also found the numerical solution employing quasi-linearization for a second-grade fluid past a stretching sheet. Ariel [2] acquired the perturbation and asymptotic solution to second-grade fluid passing through a radially stretched sheet. Ariel [3] discovered a precise solution of fluid of second-grade flowing with in a parallel channel in geometries of two different types. Tan and Masuoka [19] discussed the exact solution of the second-grade fluid in a porous half space on a heated flat plate. Fetecau and Fetecau [8] used the Fourier sine transform for cosine and sine oscillations on an infinite plate to offer the precise solution for the second-grade fluid.

Hayat et al. [10] solved the steady electrically conducting second-grade fluid flow within a porous channel using the analytical method HAM. Hayat et al. [9] used modified Darcy's law for modelling the fluid model of second-grade and analysed the effect of porous media and magnetic field for some unidirectional second-grade fluid flows. Massoudi et al. [14] studied the natural convective flow for second grade fluid with variable viscosity within parallel vertical plate applying the collocation method. Ayub and Zaman [4] derived the exact momentum equation for second-grade fluid. Kecebas and Yurusoy [12] numerically examined an incompressible unsteady modified power law fluid of second grade. Abbasbandy et al. [1] obtained the HAM solution of power-law fluid of second grade passing through an infinite porous plate and a comparison was made with a numerical solution. Rana and Latif [17] obtained an analytical solution to study three-dimensional free convective second-grade fluid flow in a porous medium subjected to constant suction and variable permeability.

Most of the above studies have adopted the analytical methods that are appropriately described for small parametric values physically and monotonous calculation is needed for any exchange in boundary conditions. The rapid convergence and flexibility for all types of physical parametric values of the employed numerical damped-Newton method motivated the present study. In this study, a fully implicit FDM is used to analyse the unsteady free convective flow of a second-grade fluid passing through a porous vertical plate with uniform suction provided close to the wall. The recent work progress is in the following way.

The flow geometry is discussed in section 2. The finite difference discretization with its numerical solution is described in section 3. Analysis of obtained results through graphical representation is done in section 4. Section 5 summarizes the conclusion..

2. Mathematical analysis

Consider the heat transfer and unsteady flow of a grade two incompressible fluid past an infinitely long porous vertical wall. Here x^*-axis is taken parallel to the vertical plate and y^*-axis is perpendicular to it. Initially, at time $t^*=0$, the plate is at rest but suddenly starts moving in direction of x^*-axis with a velocity $U(t)$ when $t^*>0$. While zero temperature is maintained at $t^*=0$, for time $t^*>0$, it is expected that the plate will be exposed to an oscillating temperature. Due to the indefinite length of the plate, the purely physical characteristics are functions of only y^* and t^*. The equation governing the flow with above assumptions is obtained as

$$\rho\left(\frac{\partial u^*}{\partial t^*}-V_0\frac{\partial u^*}{\partial y^*}\right)=\mu\frac{\partial^2 u^*}{\partial y^{*2}}+\alpha_1\left(\frac{\partial^3 u^*}{\partial y^{*2}\partial t^*}-V_0\frac{\partial^3 u^*}{\partial y^{*3}}\right)+\rho\beta g(T^*-T^*_\infty), \tag{1}$$

with conditions

$$\begin{cases} t^*\leq 0: & u^*=0, \quad \text{for every } y^*, \\ t^*>0: & u^*=U(t^*)=\frac{U^{2n+1}}{\nu^n}e^{a^*t^*}t^{*n}, \quad \text{for } y^*=0. \\ t^*>0: & u^*=0, \quad \text{for } y^*\to\infty. \end{cases} \tag{2}$$

The corresponding energy equation was obtained as

$$\rho C_p\left(\frac{\partial T'}{\partial t^*}-V_0\frac{\partial T^*}{\partial y^*}\right)=k\frac{\partial^2 T^*}{\partial y^{*2}}, \tag{3}$$

conditional to

$$\begin{cases} t^*\leq 0: \text{when} & T^*=0, \quad \forall\, y^*, \\ t^*>0: \text{when} & T^*=T^*_\infty+(T^*_w-T^*_\infty)\cos\omega^* t^*, \quad \text{at } y^*=0. \\ t^*>0: \text{when} & T^*=T^*_\infty, \quad \text{for } y^*\to\infty. \end{cases} \tag{4}$$

The joule and vicious dissipation are ignored, due to little effect on free convective flow. Furthermore, the velocity of suction indicates by $V_0>0$ and x^* and y^* are the axis of fluid velocities with u^* and v^* respectively. Here We select $U(t^*)=\frac{U^{2n+1}}{v^n}e^{a^*t^*}t^{*n}$ for computing.
We present the subsequent dimensionless quantities:

$u=\frac{u^*}{U}$, $\theta=\frac{T^*-T^*_\infty}{T^*_w-T^*_\infty}$, $y=\frac{y^*U}{\nu}$, $a=\frac{a^*\nu}{U^2}$, $t=\frac{t^*U^2}{\nu}$, $R=\frac{V_0}{U}$, $P_r=\frac{\nu\rho C_p}{k}$, $g_r=\frac{\beta g\nu(T^*_w-T^*_\infty)}{U^3}$, $\alpha=\frac{\alpha_1 U^2}{\rho\nu^2}$, $\omega=\frac{\omega^*\nu}{U^2}$.
Implementing the above non dimensional parameters in equations (1) and (2), the required governing model results as

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2 \partial t} - R\alpha\frac{\partial^3 u}{\partial y^3} + g_r\theta, \tag{5}$$

along with the boundary and initial conditions

$$\begin{cases} t \leq 0: & u = 0, \quad \forall\, y, \\ t > 0: & u = e^{at}t^n, \quad \text{for } y = 0, \\ t > 0: & u = 0, \quad \text{for } y \to \infty. \end{cases} \tag{6}$$

The governing heat flow equation so developed is represented as

$$\frac{\partial \theta}{\partial t} = R\frac{\partial \theta}{\partial y} + \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2}, \tag{7}$$

along with the boundary and initial conditions

$$\begin{cases} t \leq 0: & \theta = 0, \quad \forall\, y, \\ t > 0: & \theta = \cos\omega t, \quad \text{for } y = 0, \\ t > 0: & \theta = 0, \quad \text{for } y \to \infty. \end{cases} \tag{8}$$

3. Solution procedure

Subsequently, the PDEs are transformed into a system of coupled algebraic equation after discretising the derivatives with the help of implicit finite differences. The flow domain is divided by determining $\triangle y$ as the uniform mesh step and $\triangle t$ as time step. The grid points so generated are of the type
$(y_h, t_k) = (h\triangle y, k\triangle t)$,
$h = 0, 1, ..., N^* + 1$ and
$k = 0, 1, ..., M^*$.

The technique of minimising the PDEs (5) and (7) to a system of algebraic equations utilising a finite difference scheme(FDM) is discussed in the next subsections, and the numerical approximation solution of this system is obtained through a method called Damped-Newton[7].

3.1. Finite difference scheme

By adopting an implicit finite difference method(FDM) and a uniform space of mesh h and the time step $\triangle t$ the equations (5) and (6) are discretized, resulting in grid points of the form
$(y_h, t_k) = (h\triangle y, k\triangle t)$,
$h = 0, 1, ..., N^* + 1$ and
$k = 0, 1, ..., M^*$.

The discretized form of the velocity equation with corresponding initial and boundary conditions is obtained as

$$\frac{u_h^{k+1}-u_h^k}{\triangle t}-\frac{R}{4\triangle y}\left(u_{h+1}^{k+1}-u_{h-1}^{k+1}+u_{h+1}^{k}-u_{h-1}^{k}\right)-\frac{1}{2(\triangle y)^2}\left(u_{h+1}^{k+1}-2u_h^{k+1}+u_{h-1}^{k+1}+u_{h+1}^{k}-2u_h^k+u_{h-1}^{k}\right)$$
$$-\frac{\alpha}{\triangle y^2\triangle t}\left(u_{h+1}^{k+1}-2u_h^{k+1}+u_{h-1}^{k+1}-u_{h+1}^{k}+2u_h^k-u_{h-1}^{k}\right)+\frac{R\alpha}{2(\triangle y)^3}\Big(\left(-u_{h-2}^{k+1}+2u_{h-1}^{k+1}-2u_{h+1}^{k+1}+u_{h+2}^{k+1}\right)$$
$$+\left(-u_{h-2}^{k}+2u_{h-1}^{k}-2u_{h+1}^{k}+u_{h+2}^{k}\right)\Big)-g_r\left(\frac{\theta_h^{k+1}+\theta_h^k}{2}\right)=0, \tag{9}$$

and

$$\begin{cases} u_h^0=0, \text{where} \quad h=0,1,\dots,N^*+1,\\ u_0^k=e^{(a\triangle y\triangle t)}(\triangle y\triangle t)^n,\\ u_{N^*+1}^k=0, \text{where} \quad k=1,\dots,M^*,\end{cases} \tag{10}$$

here for the nodes $(1,h\triangle t)$, we implement $\frac{\partial^3 u}{\partial y^3}\approx\frac{1}{2h^3}\left(d_{3h}'^{k+1}+d_{3h}'^{k}\right)$.
where $d_{3h}'^{k}=-3u_{h-1}^k+10u_h^k-12u_{h+1}^k+6u_{h+2}^k-u_{h+3}^k$.
and for the nodes $(N,h\triangle t)$, we implement $\frac{\partial^3 u}{\partial y^3}\approx\frac{1}{2h^3}\left(d_{3h}''^{k+1}+d_{3h}''^{k}\right)$.
where $d_{3h}''^{k}=u_{h-3}^k-6u_{h-2}^k+12u_{h-1}^k-10u_h^k+3u_h^k$. The difference approximations for time and space used here attain second-order convergence. The discretized form of energy equation with use of exponential fitted scheme as referred in [16] with corresponding initial and boundary conditions

$$\frac{\theta_h^{k+1}-\theta_h^k}{\triangle t}-\frac{1}{P_r(\triangle y)^2}\left(\theta_{h+1}^{k+1}-2\theta_h^{k+1}+\theta_{h-1}^{k+1}\right)\frac{R\triangle y}{2}\left(\frac{1+e^{-P_rR\triangle y}}{1-e^{-P_rR\triangle y}}\right)-\frac{R}{2\triangle y}\left(\theta_{h+1}^{k+1}-\theta_{h-1}^{k+1}\right)=0, \tag{11}$$

and

$$\begin{cases}\theta_h^0=0, \text{where} \quad h=0,1,2,\dots,N^*+1, and\\ \theta_0^k=\cos(\omega\triangle y\triangle t), \text{where} \quad k=0,1,2,\dots,M^*,\end{cases} \tag{12}$$

where u_h^k,θ_h^k are approximations of velocity and temperature respectively at nodes $(h\triangle y,k\triangle t)$ for $h=0,1,2,\dots,N^*+1$, and $k=0,1,2,\dots,M^*$.

3.2. Numerical solution

The above coupled system of algebraic equations is resolved to adopt the quadratic convergence numerical scheme damped-Newton method described in [7]. The residuals R_h, $h=0,1,2,...,N^*$ is assessed alongside the components of the Jacobian Matrix $J=\left(\frac{\partial R_h}{\partial u_k}\right)$, $h=1,2,3,4\dots,N^*$ and $k=1,2,3,4\dots,M^*$ that are not zero. An appropriate initial guess for velocity leads to a convergent solution that is obtained here by solving the tri-diagonal system of equation on setting $\alpha=0$ and $g_r=0$ in (9). The advanced level temperature is obtained from (11) by means of (12). Thereafter, advanced level velocities are upgraded from a system of equation $Jh=-R_h$ using suitable damping. This coupled procedure continues upto the maximum iteration limit N^*, where the difference between two consecutive approximation solutions is assumed be lower than a necessary tolerance value ϵ. Because of a sufficiently accurate initial guess, the results obtained here are correct up to five decimal places.

4. Result analysis

The above system of coupled algebraic equation is numerically solved by adopting a finite difference scheme implicitly and then simulated using MATLAB programming. The results so obtained are depicted through graphs. If not otherwise specified, the flow parameter values $\omega = 0.1$, $dt = 0.01$, and $a = 0.5$, $hl = 0.01$ is kept fixed for all plots. Results are examined for various velocity and temperature field parameters, primarily for two situations., $n = 0.5$ (variable acceleration), and $n = 1$ (constant acceleration).

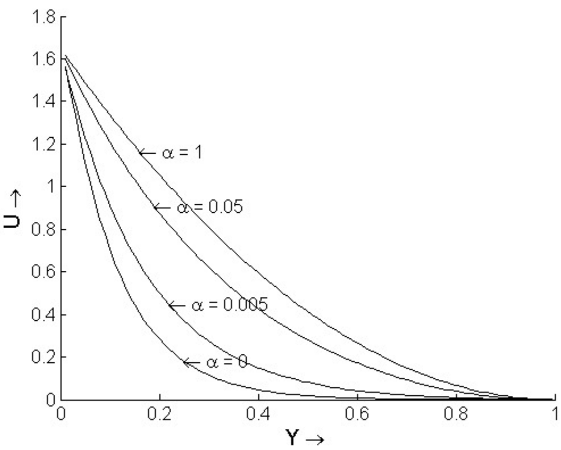

Figure 1: *Variation of* α *when* $g_r = 10, R = 9, P_r = 10.$

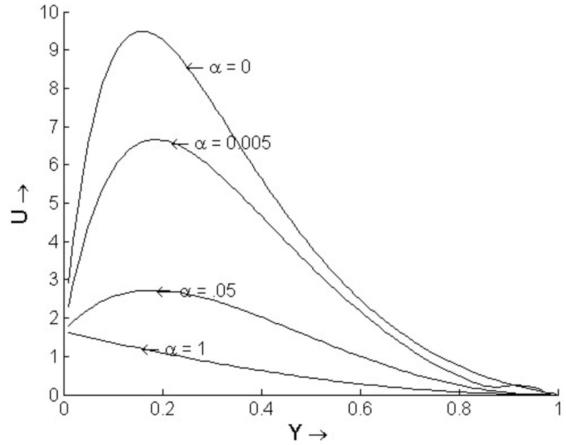

Figure 2: *Variation of* α *when* $g_r = 10, R = 9, P_r = 0.5.$

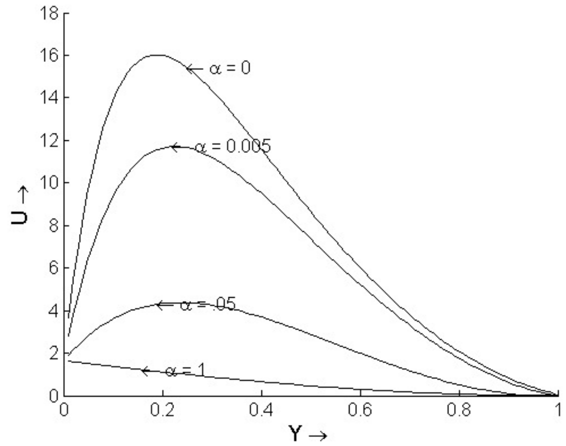

Figure 3: *Variation of* α *when* $g_r = 10, R = 9, P_r = 0.05.$

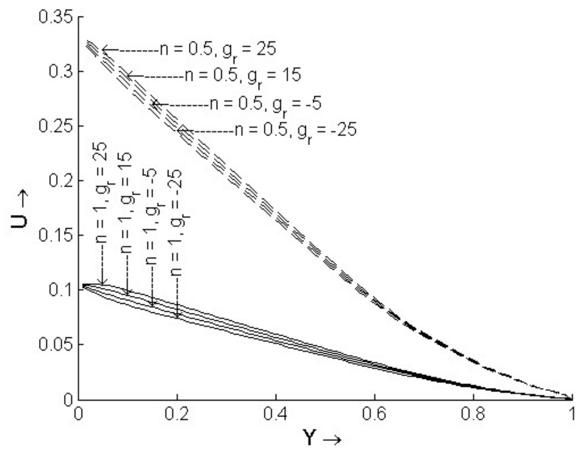

Figure 4: *Variation of* g_r *when* $(n = 1, 0.5)$, $\gamma = 0, \alpha = 1, R = 10, P_r = 10.$

ISSN: 2956-6053

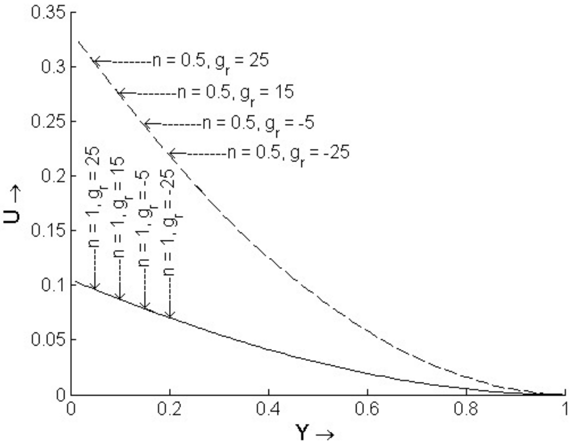

Figure 5: *Variation of g_r when,* $\gamma = 0, \alpha = 1, R = 10, P_r = 10.$

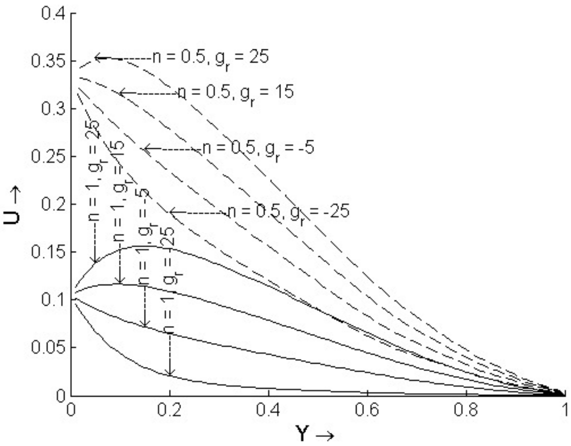

Figure 6: *Variation of g_r when,* $\alpha = 1, R = 10, P_r = -10, \gamma = 0,$

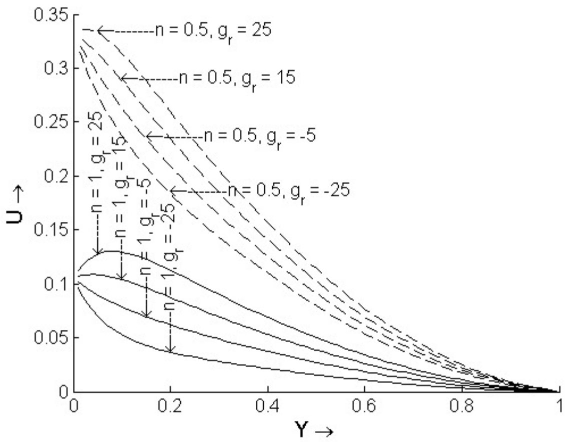

Figure 7: *Velocity variation of g_r .*

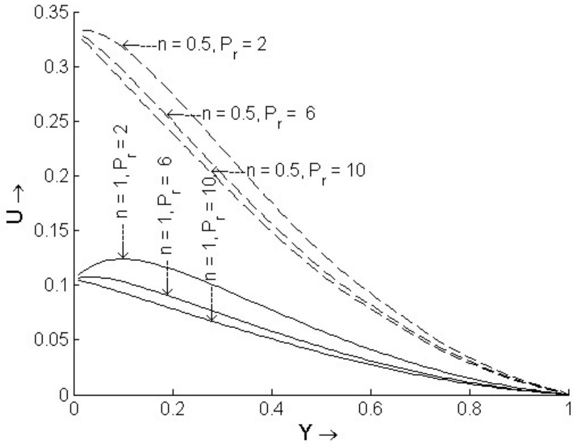

Figure 8: *Velocity variation of P_r.*

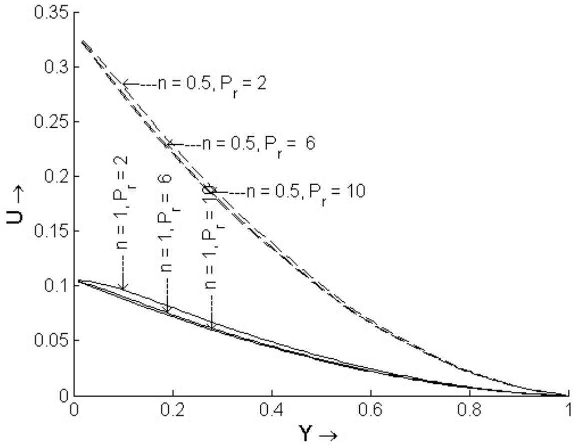

Figure 9: *Velocity variation of P_r.*

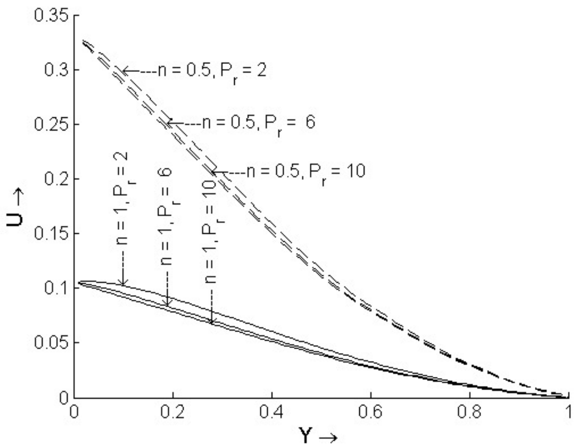

Figure 10: *Velocity variation of P_r.*

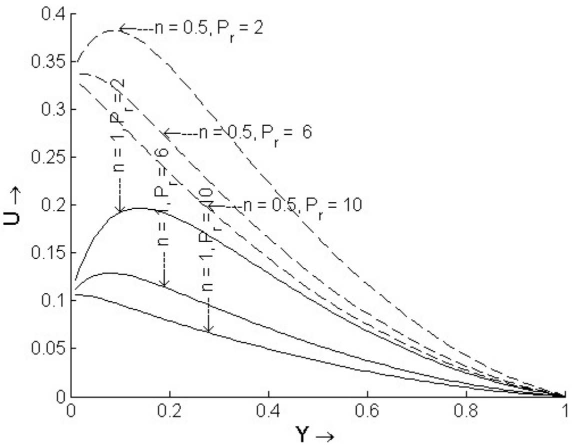

Figure 11: *Velocity variation of P_r.*

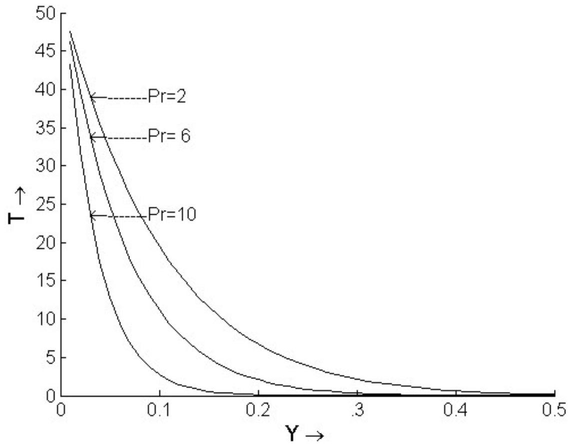

Figure 12: *Temperature variation of P_r.*

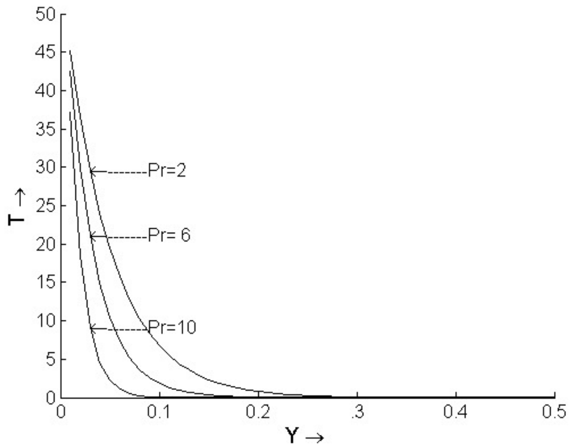

Figure 13: *Temperature variation of P_r.*

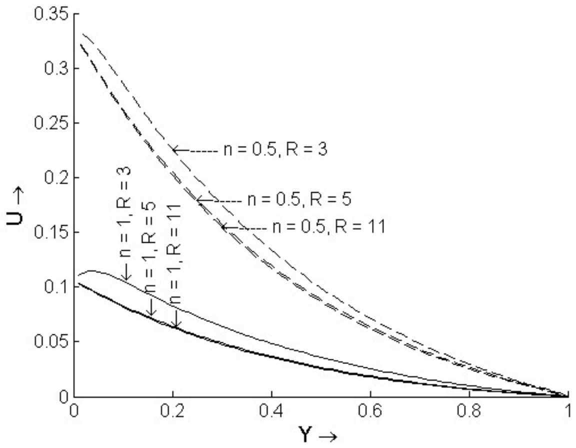

Figure 14: *Velocity variation of R.*

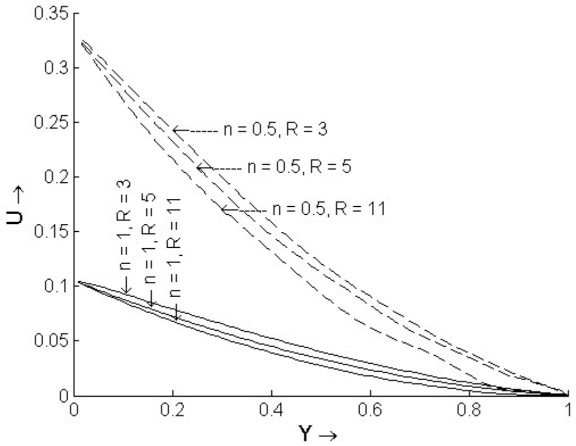

Figure 15: *Velocity variation of R.*

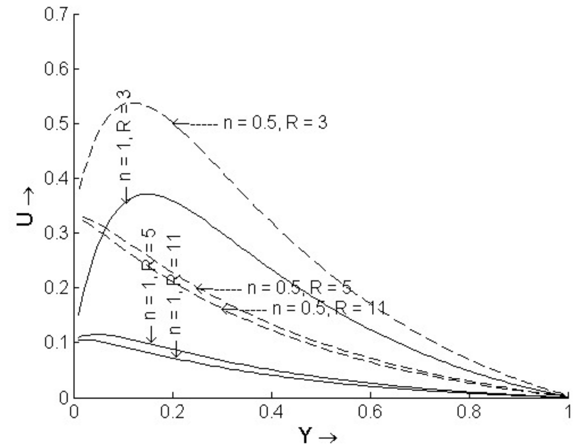

Figure 16: *Velocity variation of R.*

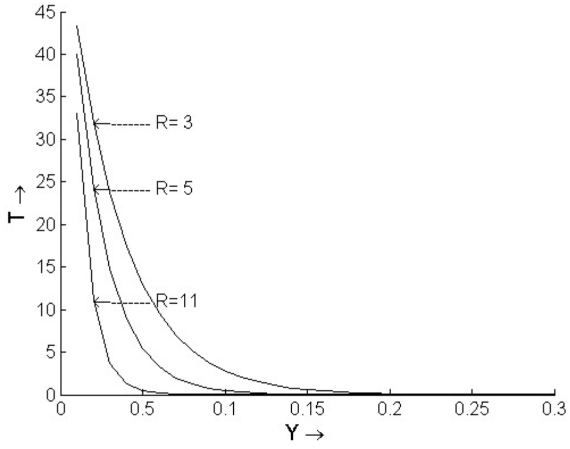

Figure 17: *Temperature variation of R.*

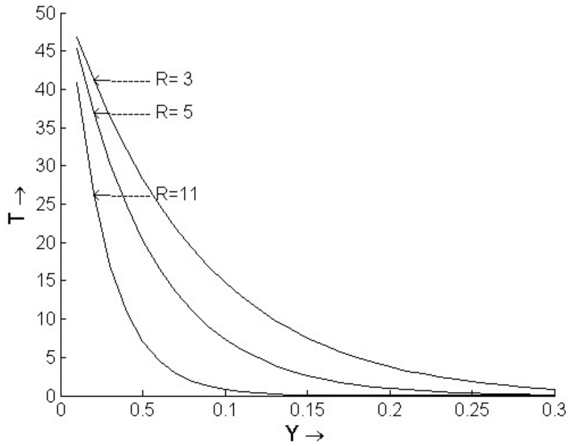

Figure 18: *Temperature variation of R.*

Figure 1 - 3 demonstrates the effect of values for the second-grade elastic factor α on the velocity profile, on variation of different emerging flow parameters. The velocity profile is found to be increasing with increase of elastic parameter value α when $P_r = 10$, but when P_r value is reduced to 0.3, a reverse effect is experienced in the profile. An exponential growth of velocity profile is observed near the plate surface and gradually boundary level thickness decreases away from the plate. Further reducing P_r values it is analysed that velocity profile decreases at a faster rate than the previous case.

Figure 4 - 7 describes the effect of grashof number g_r on velocity profile for various flow parameters, both for constant and variable acceleration case. Because of free convention, positive grashof number means cooling of the vertical plate and negative grashof number represent heating of the plate. In all situations, it is shown that the velocity profile rises as the g_r values climb, which is due to a reduction in dynamic viscosity as g_r increases, and that intensifies the velocity flow. Maximum velocity is attained near the wall which gradually seems to decreases. But this increasing behaviour diminishes as we go on to increase the value of suction parameter to $R = 8$ that leads to an increase in internal viscous forces of the fluid. In addition to it, as we reduce the P_r values diffusive heat of fluid decreases which allows more fluid to pass, hence a profound increasing flow behaviour is observed.

Figure 8 - 13 expresses the effect of prandtl number P_r on flow and energy profiles on varying different physical flow parameters. It is perceived that velocity profile initially rises exponentially close to the wall surface and then a steady decrease through out the flow domain is noticed. But when the suction parameter R is slightly increased, the momentum boundary layer thickness declines significantly due to rise of additional inertial forces within fluid. A similar behaviour is observed as we increase α values. With reduction in the elastic parameter values, an instant hike of velocity profile occurs near the plate which decreases remarkably. Temperature profile is found to be a decreasing function of P_r both for $g_r < 0$ and $g_r > 0$.

Figure 14 - 18 depicts the impact of the suc-

tion parameter R in the scenarios of $n = 1, 0.5$. It is witnessed that as R values rise, the velocity profile across the whole flow domain decreases. This effect is slowly diminished far away from the surface of the wall. As the elastic parameter values increase, a pronounced decreasing behaviour is seen through out the flow domain for both the cases i.e when either a constant or variable acceleration is suddenly applied to the vertical plate. As the suction parameter value increases, the temperature profile similarly drops.

5. Conclusion

In present work a numerical study of heat transfer and flow of a free convective second grade fluid passed through porous plate vertically is analysed for different non-dimensional flow parameters. The quadratic convergence numerical scheme adopted here to obtain appropriate stable results for small and large flow parameter values and overcomes persistent assessment of the entire problem for any modification in boundary conditions. Some significant findings of present analysis are noted hereby:

- For large P_r values velocity profile increases with increase in second-grade elastic parameter, but has a contrary effect on velocity for smaller P_r values.
- Velocity profile seems to increase for $g_r < 0$ and $g_r > 0$ for both smaller and comparatively larger P_r values. But the effect is insignificant when suction parameter value is more than 8.
- As P_r values increases, with an increase in the suction parameter, the velocity profile slows down even further, however as the elastic parameter value is decreased, a significant decline is observed.
- Both velocity and temperature field decelerates with an increase in R values, while other flow parameter values are also altered.

Nomenclature:

1. ρ = Density.
2. g = Acceleration due to gravity.
3. μ = Dynamic viscosity.
4. ν = Kinematic viscosity.
5. V_0 = Velocity of suction.
6. T^*_∞ = Temperature of a fluid at a distance from a plate.
7. t^* = Time.
8. C_p - Cpecific heat under a certain pressure.
9. k = The fluid's thermal conductivity.
10. β =Volume expansion coefficient.
11. u^*, U = velocities of the fluid.
12. g_r = Thermal Grashof number.
13. R = Suction parameter.
14. α_1 = Material constant with dimension ML^{-1}.
15. P_r = Prandtl number.
16. α = Elastic parameter.
17. ω = Frequency of oscillation.
18. a' = Constant having dimension T^{-1}.

ISSN: 2956-6053

References

[1] Abbasbandy, S., Yurusoy, M. and Gulluce, H., (2014), Analytical solutions of non-linear equations of power law fluids of second grade over an infinite porous plate, Mathematical and Computational Applications, 19(2), pp.124-133.

[2] Ariel, P. D., (2001), Axisymmetric flow of a second grade fluid past a stretching sheet, International Journal of Engineering Science, 39(5), pp.529-553.

[3] Ariel, P. D., (2002), On exact solution of flow problems of a second grade fluid through two parallel porous walls, International Journal of Engineering Science, 40(8), pp.913-941.

[4] Ayub, M. and Zaman, H., (2010), Complete derivation of momentum equation for second grade fluid, Journal of Mathematics and Computer Science, 1(1), pp.33-39.

[5] Bandelli, R. and Rajagopal, K. R., (1995), Start up flows of second grade fluids in domains with one finite dimension, International Journal of Non-linear Mechanics, 30(6), pp.817-839.

[6] Beavers, G. S. and Joseph, D. D., (1975), The rotating rod viscometer, Journal of Fluid Mechanics, 69(3), pp.475-511.

[7] Conte, S. D. and De Boor, C., (1980), Elementary Numerical Analysis an Algorithmic Approach, McGraw-Hill Inc., New York.

[8] Fetecau, C. and Fetecau, C., (2005), Starting solution for some unsteady unidirectional flows of second grade fluid, International Journal of Engineering Science, 43(10), pp.781-789.

[9] Hayat, T., Khan, I., Ellahi, R. and Fetecau, C., (2008), Some MHD flows of a second grade fluid through the porous medium, Journal of porous media, 11(4), pp. 389-400.

[10] Hayat, T., Ahmed, N., Sajid, M. and Asghar, S., (2007), On the MHD flow of a second grade grade fluid in a porous channel, Computers and Mathematics with Applications, 54 (3), pp.407-414.

[11] Jain, M. K., (1984), Numerical Solution of Differential equations, Wiley Eastern, New Delhi.

[12] Keceba, A. and Yurusoy, M., (2011), Numerical solution of unsteady boundary layer equations for a generalized second grade fluid, Journal of Theoretical and Applied mechanics, 49(1), pp.71-82.

[13] Massoudi, M. and Maneschy, C. E., (2004), Numerical solution to the flow of a second grade fluid over a stretching sheet using the method of quasi-linearization, Applied Mathematics and computation, 149 (1), pp.165-173.

[14] Massoudi, M., Vaidya, A. and Wulandana, R., (2008), Natural convection flow of a generalized second grade fluid between two vertical walls, Nonlinear Analysis: Real World Application, 9(1), pp.80-93.

[15] Mohapatra, P., Padhy, S. and Tripathy, S., (2001), Finite difference analysis of the flow of a second order liquid past an infinite vertical plate in the presence of a magnetic field, International Journal of Applied mechanics and Engineering, 6(1), pp.71-90.

[16] Morton, K. W., (1996), Numerical solution of Convection-Diffusion problems, Chapman and Hall, London.

[17] Rana, M. A. and Latif, A., (2019), Three dimensional free convective flow of a second grade fluid through a porous medium with periodic permeability and heat transfer, Boundary value problems, 44, pp.1-19.

[18] Rivlin, R. S. and Erickson, J. L., (1955), Stress deformation relation for isotopic materials, Journal of Rational Mechanics and Analysis, 4, pp.323-425.

[19] Tan, W. C. and Masuoka, T., (2005), Stokes' first problem for second grade fluid in a porous half-space with heated boundary, International Journal of Non-Linear mechanics, 40(4), pp.515-522.

[20] Teipel, I., (1998), Stagnation point flow of a non-Newtonian second order fluid, Transactions of the Canadian Society for Mechanical Engineering, 12(2), pp.57-61.

[21] Truesdell, C. and Noll, W., (1992), The non linear field theories of mechanics, Springer, 2nd Ed.