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Improving Decision-Making Under Uncertainty: A Comparative Study of Fuzzy Set Extensions

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Abstract

Fuzzy sets have revolutionized decision-making by providing a mathematical tool for modeling uncertainty and imprecision. However, traditional fuzzy sets may not be sufficient in certain situations, leading to the development of extensions such as Type-2 fuzzy sets, Intuitionistic fuzzy sets, and Type-2 intuitionistic fuzzy sets. This paper provides an overview of these sets, comparing and contrasting them using operations of union, intersection, and distance measures. Additionally, a new distance measure is proposed for Type-2 intuitionistic fuzzy sets, which is demonstrated with a numerical example. Our novel distance measure proves to be the best tool for decision making problems containing uncertainty and the result is compared with the existing distance measures. By understanding the properties and applications of these fuzzy sets, informed decisions can be made in real-world situations with uncertainty and imprecision.

Keywords: Fuzzy set, Type-2 Fuzzy set, Intuitionistic Fuzzy set, Type-2 Intuitionistic Fuzzy set, Distance measure

1. Introduction

L.A. Zadeh [5] developed fuzzy set(FS) theory in response to the requirement to represent the activity of modelling in the human mind, which must take into account subjective and imprecise elements. Its key idea is membership grade(M-G), A member is either in or out of a subset according to conventional set theory. A proposition is either true or false in boolean logic. Information by its nature contains uncertainty, we make decisions in environments with various types of uncertainty in many scientific and industrial applications. Currently, the majority of decision-making procedures involve acquiring and processing information, much of which is noisy, fragmented, inconsistent, or all of the above. As a result, The models that explain the real world must be supplemented by appropriate uncertainty representations. With the introduction of soft computing approaches, many powerful tools in the field of computational intelligence, such as type-1 fuzzy logic, evolutionary algorithms, hybrid intelligent systems, and neural networks, were produced. [4, 24].

An extension of the ordinary FS, or type-1 fuzzy set(T1FS) is the (Type-2 fuzzy sets) T2FS. T2FSs could be referred to as a "fuzzy-fuzzy set" because the M-Gs are ambiguous and the domain of T2FSs is T1FS instead of crisp value. Zadeh [35, 36, 37] introduced the idea of T2FS. Mendel [23] provided overviews of T2FSs. Since T2FSs are a specific case of ordinary FSs and interval-valued fuzzy sets(IVFS), Takac [30] suggested that T2FSs are very useful in situations where there are more uncertainties. From the perspectives of type reduction and the centroid, Kundu et al.[17] gave a fixed charge transportation problem with type-2 fuzzy parameters. Both Dubois, Prade [6] and Mizumoto, Tanaka [19, 20] looked at the logical behaviour of T2FS. Later, a large number of scholars conducted extensive research on T2FS, theoretical and numerous application areas [12, 13, 15, 18].

The (intuitionistic fuzzy sets) IFS developed by Atanassov [2] that can be expressed in terms of the degrees of membership, and degree of non-membershipis a more generalised variant of the FS. The study of problems like decision-making by utilising IFSs, however has attracted more attention [25]. In order to address the issue of students satisfaction with university instruction, Marasini et al. [27] used an IFS technique that may take into consideration two sources of uncertainty: one connected to items and the other to subjects. Dan et al.[10] Present the generalised (Type-2 intuitionistic fuzzy set) T2IFS, whose type-1 membership is the conventional fuzzy membership and whose type-2 comprises both membership and non-membership as the IFS. Singh.S and Garg.H [28] proposed a multi criteron decision making problem by providing a distance measure for T2IFS. Some t-conorm-based distance measures and knowledge measures for Pythagorean fuzzy sets with their application in decision-making was given by Ganai. A. H.[42]. A Multicriteria decision-making based on distance measures and knowledge measures of Fermatean fuzzy sets given by Ganie. A. H. [40]. A Generalized hesitant fuzzy knowledge measure with its application to multi-criteria decision-making is given by Singh, S. and Ganie, A. H. [41]. Almulhim, T. and Barahona, I. [43] gave an extended picture fuzzy multicriteria group decision analysis with different weights: A case study of COVID-19 vaccine allocation.

Fuzzy sets have transformed decision making by providing a mathematical tool for modeling uncertainty and imprecision. However, traditional fuzzy sets may not be adequate in certain situations, leading to the development of type-2 fuzzy sets, which introduce a third dimension to membership functions to allow for more precise definitions of uncertainty. Different extensions of fuzzy sets exist to make them more manageable, and understanding their properties is crucial for selecting the most suitable set for specific conditions. T1FS, T2FS, IFS, and T2IFS are sets examined for their properties, with numerical examples provided for comparison. Furthermore, a new distance measure is proposed for T2IFSs, demonstrating its significance with an example. By grasping the diverse properties and applications of these fuzzy sets, informed decisions can be made in real-world situations with uncertainty and imprecision.

This paper is divided into several sections to help you understand and compare different existing fuzzy sets. In section **??**, we'll cover the preliminaries and basic concepts to give you a solid foundation. Then in section 3 , we'll compare different fuzzy sets using the operations of union and intersection. We'll explore their similarities and differences, helping you make informed decisions for your specific needs. section 4 proposes a new distance measure for T2IFS, accompanied by a numerical example to compare the results. Finally, in section 5, we'll draw our conclusions and tie it all together.

2. Preliminaries and Basic Concepts

2.1. Fuzzy set (FS)

Definition 1. *[33] A FS J in S is a set of an ordered pair if S is a collection of elements denoted generally by s:*

$$J = \{(s, \mu_J(s))|s \in S\} \tag{1}$$

where $\mu_J(s)$ is called M-F of FS J in S and its value lies in between closed interval $[0, 1]$.

2.2. Operation on Fuzzy sets

The following operations for FSs are defined by [33] as generalisations of crisp sets and crisp statements in his first paper.

Definition 2. *Intersection [logical and]: The following M-F is used to describe the intersection of the FSs J and K*

$$\mu_{J\cap K}(s) = Min\{(\mu_J(s), \mu_K(s))\forall s \in S \tag{2}$$

Definition 3. *Union [exclusive or]: The union's M-F is described as*

$$\mu_{J\cup K}(s) = Max\{(\mu_J(s), \mu_K(s))\ \ \forall s \in S \tag{3}$$

Definition 4. *Complement (negation): The following is a definition of the complement's membership function:*

$$\mu_J(s) = 1 - \mu_J(s)\ \ \forall s \in S \tag{4}$$

Later, the above defined definitions were expanded. Both the "logical and" (intersection) and the "inclusive or" (union) can be modelled as t-norms [3, 8, 9, 11, 16, 22, 32, 39]. Both kinds are associative, commutative, and monotonic. Below is a compilation of typical dual pairs of nonparameterized t-norms and t-conorms:

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Definition 5. *Drastic product:*

$$t_W(\mu_J(s), \mu_K(s)) = \begin{cases} Min\{(\mu_J(s), \mu_K(s))\} & if \;\; Max\{(\mu_J(s), \mu_K(s))\} = 1 \\ 0 & otherwise\} \end{cases} \tag{5}$$

Definition 6. *Drastic sum:*

$$S_W(\mu_J(s), \mu_K(s)) = \begin{cases} Max\{(\mu_J(s), \mu_K(s))\} & if \;\; Min\{(\mu_J(s), \mu_K(s))\} = 0 \\ 1 & otherwise\} \end{cases} \tag{6}$$

Definition 7. *Bounded difference:*

$$t_1(\mu_J(s), \mu_K(s)) = Max\{0, \mu_J(s) + \mu_K(s) - 1\} \tag{7}$$

Definition 8. *Bounded sum:*

$$s_1(\mu_J(s), \mu_K(s)) = Min\{1, \mu_J(s) + \mu_K(s)\} \tag{8}$$

Definition 9. *Einstein product:*

$$t_{1.5}(\mu_J(s), \mu_K(s)) = \frac{\mu_J(s) \cdot \mu_K(s)\}}{2 - [\mu_J(s) + \mu_K(s) - \mu_J(s) \cdot \mu_K(s)]} \tag{9}$$

Definition 10. *Einstein sum:*

$$s_{1.5}(\mu_J(s), \mu_K(s)) = \frac{\mu_J(s) + \mu_K(s)\}}{1 + \mu_J(s) + \mu_K(s)} \tag{10}$$

Definition 11. *Hamachar product:*

$$t_{2.5}(\mu_J(s), \mu_K(s)) = \frac{\mu_J(s) \cdot \mu_K(s)\}}{\mu_J(s) + \mu_K(s) - \mu_J(s) \cdot \mu_K(s)} \tag{11}$$

Definition 12. *Hamachar sum:*

$$s_{2.5}(\mu_J(s), \mu_K(s)) = \frac{\mu_J(s) + \mu_K(s) - 2\mu_J(s) \cdot \mu_K(s)\}}{1 - \mu_J(s) \cdot \mu_K(s)} \tag{12}$$

Definition 13. *Minimum:*

$$t_3(\mu_J(s), \mu_K(s)) = min\{\mu_J(s), \mu_K(s)\} \tag{13}$$

Definition 14. *Maximum:*

$$s_3(\mu_J(s), \mu_K(s)) = max\{\mu_J(s), \mu_K(s)\} \tag{14}$$

The above defined operators have been ordered as follows:

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$$t_w \leq t_1 \leq t_{1.5} \leq t_2 \leq t_{2.5} \leq t_3 \tag{15}$$

$$s_3 \leq s_{2.5} \leq s_2 \leq s_{1.5} \leq s_1 \leq s_w \tag{16}$$

The operations defined above are not valid for T2FSs because T2FSs contain type-2 membership function so extension principle is defined to deal with the operations for T2FSs.

2.3. Type-2 Fuzzy set (T2FS)

Definition 15. *T2FS [21] is defined as the extension of ordinary FS that is T1FS and is characterised by Type-2 membership function $\mu_{\bar{Z}}(s,u)$. Let S be a fixed universe a T2FS $\bar{Z} \subseteq S$ is defined mathematically as*

$$\bar{Z} = (s, u, \mu_{\bar{Z}}(s,u)) \ \ |s \in S, u \in j_s \subseteq [0,1]$$

in which $0 \leq \mu_{\bar{Z}}(s,u) \leq 1$. It can also be written as

$$\bar{Z} = \int_{s\in S} \mu_{\bar{Z}}(s)/s \quad |s \in S, u \in j_s \subseteq [0,1] = \int_{s\in S} [\int_{u\in j_s} (g_s(u)/u)]/s$$

Where $\mu_{\bar{Z}}(s) = \int_{u\in j_s}(g_s(u)/u)$ is the grade of membership,$g_s(u) = \mu_{\bar{Z}}(s,u)$ is named as secondary membership function(S-MF) where u is primary membership function(P-MF) of $\bar{Z}$ and j_s is called P-MF of S.

Definition 16. *Footprint of Uncertainty(FOU) [26] actually for T2FS we are having 3-D structure which becomes very difficult for calculation so we take the base of 3rd dimension to calculate the values which is called FOU. It can be defined as the union of all P-MF that is*

$$FOU(Z) = \cup_{s\in S}(j_s) \tag{17}$$

Example 1. *Let "Young" be the set defined by T2FS $\bar{E}$ and the P-MF of $\bar{E}$ be "Youthness„ and S-MF be degree of "Youthness". Let $T = \{7, 9, 13\}$ be the car set having primary membership at point T respectively. $j_7 = \{0.7, 0.8, 0.9\}, j_9 = \{0.5, 0.6, 0.7\}$ and $j_{13} = \{0.3, 0.4, 0.5\}$ then S-MF of point 7 is $\bar{\mu}_{\bar{E}}(7,u) = \{(0.8/0.7) + (0.6/0.8) + (0.5/0.9)\}$ that is $\bar{\mu}_{\bar{E}}(7, 0.7) = 0.8$ is the secondary membership grade of 7 with respect to 0.7 similarlaly $\bar{\mu}_{\bar{E}}(9,u) = \{(0.7/0.5) + (0.6/0.6) + (0.5/0.7)\}$ and $\bar{\mu}_{\bar{E}}(13,u) = \{(0.8/0.3)+(0.7/0.4)+(0.4/0.5)\}$ then discrete T2FS can be defined accordingly $\bar{E} = \{(0.8/0.7)+(0.6/0.8)+(0.5/0.9)\}/7 + \{(0.7/0.5) + (0.6/0.6) + (0.5/0.7)\}/9 + \{(0.8/0.3) + (0.7/0.4) + (0.4/0.5)\}/13$*

Definition 17. *Extension Principle : The extension principle is one of the most fundamental ideas in FS theory that can be used to apply clear mathematical ideas to FSs. It was already suggested in Zadeh's initial contribution in its simplest form. Modifications have been suggested in the interim. Zadeh, Dubois, and Prade [25, 27, 28] provided the following definition of the extension principle:*

Let $E_1, E_2..., E_r$ be r fuzzy sets in $S_1, S_2..., S_r$ and S be the Cartesian product of universes S = $S_1 \times ..., \times S_r$, respectively. , where f is a mapping from S to a universe T. t = $f(s_1, ..., s_r)$. We can then define a fuzzy set F in T by using the extension principle concept

$$\bar{F} = \{t, \mu_{\bar{F}}(t)|t = f(s_1, ..., s_r), (s_1, ..., s_r) \in S\} \tag{18}$$

$$\mu_{\bar{F}}(t) = \begin{cases} \{sup_{(s_1,...,s_r)\in f^{-1}(t)} min\{\mu_{\bar{E}_1(s_1)}, ..., \mu_{\bar{E}_r(s_r)}\} \\ if \quad f^{-1}(t) \neq 0 \\ 0 \qquad\qquad\qquad otherwise\} \end{cases} \tag{19}$$

Where f^{-1} is the inverse of f

if we put r=1 then the extension principle is reduced to

$$\bar{F} = \{f(\bar{E}) = \{(t, \mu_{\bar{F}}(t))|t = f(s), s \in S\} \tag{20}$$

where

$$\mu_{\bar{F}}(t) = \begin{cases} \{sup_{(s)\in f^{-1}(t)} min\{\mu_{\bar{E}(s)}\} & if \quad f^{-1}(t) \neq 0 \\ 0 & otherwise\} \end{cases} \tag{21}$$

2.4. Intuitionistic Fuzzy set

Definition 18. *An IFS is a set which is having both a M-F and a N-MF, as opposed to a classical fuzzy set, which only has a M-F. An object of the form is what Atanassov [1] defines as an IFS J in S.*

$$J = \{s, \mu_J(s), \nu_J(s) : s \in S, \mu_J(s) \in [0, 1], \nu_J(s) \in [0, 1]\} \tag{22}$$

where as $\mu_J(s) : S \to [0, 1]$ and $\nu_J(s) : S \to [0, 1]$ is called as degree of membership and degree of non-membership respectively such that $0 \leq \mu_J(s) + \nu_J(s) \leq 1 \forall s \in S$

Example 2. *Let "Young" be the set defined by IFS J. The degree of "Youthness" and "Adultness" are membership and non-membership respectively. Let $T = \{11, 14, 16\}$ and the M-G of the point 11 be $\mu_P(12) = \{0.7, 0.8, 0.9\}$ and the non-membership grade(N-MG) of point 11 is $\nu_P(11) = \{0.1, 0.2, 0.0\}$ similarly $\mu_P(14) = \{0.5, 0.6, 0.7\}, \nu_P(14) = \{0.4, 0.3, 0.1\}$ and $\mu_P(16) = \{0.4, 0.5, 0.6\}, \nu_P(16) = \{0.5, 0.4, 0.2\}$*

2.5. Type 2 intuitionistic Fuzzy set(T2IFS)

Definition 19. *[28] A T2IFS J in the universe of discourse S is set of pairs $\{s, \mu_J(s), \nu_J(s)\}$ where s is the element of T2IFS, $\mu_J(s)$ and $\nu_J(s)$ are called grades of the membership and non-membership respectively* *defined in the interval [0,1] as*

$$\mu_J(s) = \int_{s \in j_s^1} (g_s(u)/u), \quad \nu_J(s) = \int_{s \in j_s^2} (h_s(v)/v) \tag{23}$$

Where $g_s(u)/u$ and $h_s(v)/v$ are termed as S-MF and secondary non-membership function(S-NMF). In addition μ_J,ν_J denotes the P-MF and primary non-membership functions (P-NMF) and j_{s^1} and j_{s^2} are named as the P-MF and P-NMF of S, respectively. In other words, T2IFS J is defined in the universe of discourse as

$$J = \{(s, u_J, v_J), g_{sj}(u_J), h_{sj}(v_J) | s \in S, u_J \in j_{s^1}, v_J \in j_{s^2}\} \tag{24}$$

Where the element of the domain $(s, (u_J, v_J))$ called as P-MF (u_J) and P-NMF (v_J) of $s \in S$ where $g_{sj}(u_J)$ and $h_{sj}(v_J)$ S-MF and S-NMF respectively.

3. Comperative analysis on different types of fuzzy sets

3.1. Comparison on the basis of operation

In order to make comparison we take few fuzzy sets into account, ordinary FS or T1FS, T2FS, IFS and T2IFS we define union and intersection for these defined sets

3.2. Union and Intersection for T1FS

let J and K be two fuzzy sets then their union and intersection is defined as follows

Union:

$J \cup K = max\{\mu_J(s), \mu_K(s)\}$
where $\mu_J(s)$ and $\mu_K(s)$ are the membership values of FS J and K.

Example 3. *Let J={s,0.8} and K={s,0.7} then* $J \cup K = max\{0.8, 0.7\} \implies J \cup K = 0.8$

Intersection:

$J \cap K = min\{\mu_J(s), \mu_K(s)\}$
where $\mu_J(s)$ and $\mu_K(s)$ are the membership values of FS J and K.

Example 4. *Let J={s,0.8} and K={s,0.7} then* $J \cup K = min\{0.8, 0.7\} \implies J \cap K = 0.7$

3.3. Union and intersection for T2FS

Let μ_J and μ_K are two T2FS

Intersection:

$\mu_J = \{s, \mu_J(s)\}$ and $\mu_K = \{s, \mu_K(s)\}$
where $\mu_J(s) = \{u_i, \mu_{ui}(s)\}$
$\mu_K(s) = \{v_j, \mu_{vj}(s)\}$
by extension principle intersection is defined as

$$\mu_{J\cap K}(s) = \{z, \mu_{J\cap K}(z)| \;\; z = \min\{u_i, v_j\}\} \tag{25}$$

where $\mu_{J\cap K}(z) = \sup_{z=\min(u_i,v_j)} \min\{\mu_{ui}, \mu_{vj}\}$.

Union:

$$\mu_{J\cup K}(s) = \{z, \mu_{J\cup K}(z)| \;\; z = \max\{u_i, v_j\}\} \tag{26}$$

where $\mu_{J\cup K}(z) = \sup_{z=\max(u_i,v_j)} \min\{\mu_{ui}, \mu_{vj}\}$.

Example 5. *Let J be a small integer and K be an integer. Find $\mu_{J\cap K}(s)$ at s=3*

Table 1

i	u_i	μ_{ui}	v_j	μ_{vj}
1	0.8	1	1	1
2	0.7	0.5	0.8	0.5
3	0.6	0.4	0.7	0.3

$J = \{s, \mu_J(s)\}$ *at s=3*
$\mu_J(s) = \{(u_1, \mu_{u1}), (u_2, \mu_{u2}), (u_3, \mu_{u3})\}$
={(0.8,1), (0.7,0.5), (0.6,0.4)}
similarly
$\mu_K(s) = \{(v_1, \mu_{v1}), (v_2, \mu_{v2}), (v_3, \mu_{v3})\}$
={(1,1), (0.8,0.5), (0.7,0.3)}

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Table 2

u_i	v_j	$\min(u_i, v_j)$	$\mu_{ui}(3)$	$\mu_{vj}(3)$	$\min(\mu_{ui}(3), \mu_{vj}(3))$
0.8	1	0.8	1	1	1
0.8	0.8	0.8	1	0.5	0.5
0.8	0.7	0.7	1	0.3	0.3
0.7	1	0.7	0.5	1	0.5
0.7	0.8	0.7	0.5	0.5	0.5
0.7	0.7	0.7	0.5	0.3	0.3
0.6	1	0.6	0.4	1	0.4
0.6	0.8	0.6	0.4	0.5	0.4
0.6	0.7	0.6	0.4	0.3	0.3

$\mu_{J\cap K}(s) = \sup_{z=0.8}\{1, 0.5\} = 1$
$\sup_{z=0.7}\{0.3, 0.5, 0.5, 0.3\} = 0.5$
$\sup_{z=0.6}\{0.4, 0.4, 0.3\} = 0.4$

3.4. Union and Intersection for IFS

let J and K be two IFSs then we define

Union:

$$J \cup K = \max\{\mu_J(s), \mu_K(s)\}, \min\{\nu_J(s), \nu_K(s)\} \tag{27}$$

Intersection:

$$J \cap K = \min\{\mu_J(s), \mu_K(s)\}, \max\{\nu_J(s), \nu_K(s)\} \tag{28}$$

Example 6. *Let we have two IFS defined as*
J={s,0.6,0.4}   and K={s,0.7,0.2} then

$$J \cup K = \max\{0.6, 0.7\}, \min\{0.4, 0.2\} = \{0.7, 0.2\}$$

3.5. Union and Intersection for T2IFSs

lets consider two T2IFS J and K

$$J = \int_{s\in S}\left(\int_{u\in i_s^u}(\mu_J(s,u), \nu_J(s,u))/u\right)/S$$

and

$$K = \int_{s \in S} \left(\int_{v \in i_s^v} (\mu_K(s,v), \nu_K(s,v))/v \right) /S$$

where $i_s^u \subseteq [0,1]$ and $i_s^v \subseteq [0,1]$ are domains for S-MF respectively. Then we define union for J and K as:

$$J \cup K = \int_{s \in S} \frac{\frac{\left(\int_{v \in i_s^w} (\mu_{J\cup K}(s,w), \nu_{J\cup K}(s,w)\right)}{w}}{S}, i_s^u \cup i_s^v = i_s^w \subseteq [0,1]$$

where

$$\mu_{J\cup K}(s) = \phi \left(\int_{u \in i_s^u} (\mu_J(s,u))/u, \int_{v \in i_s^v} (\mu_K(s,v))/v \right)$$

by using extension principle, we obtain

$$\mu_{J\cup K}(s,w) = \int_{u \in i_s^u} \int_{v \in i_s^v} \left(\mu_J(s,u) \wedge \mu_K(s,u)\right) / \phi(u,v),$$

where $\phi(u,v)$ is t-conorm of u and v

$$\mu_{J\cup K}(s,w) = \int_{u \in i_s^u} \int_{v \in i_s^v} \left(\mu_J(s,u) \wedge \mu_K(s,u)\right) / (u \vee v),$$

similarly

$$\nu_{J\cup K}(s,w) = \int_{u \in i_s^u} \int_{v \in i_s^v} (\nu_J(s,u) \vee \nu_K(s,u))/(u \vee v),$$

Intersection for J and K is defined as:

$$J \cap K = \int_{s \in S} \frac{\frac{\left(\int_{v \in i_s^w} (\mu_{J\cap K}(s,w), \nu_{J\cap K}(s,w))\right)}{w}}{S}, i_s^u \cup i_s^v = i_s^w \subseteq [0,1]$$

where

$$\mu_{J\cap K}(s,w) = \int_{u \in i_s^u} \int_{v \in i_s^v} (\mu_J(s,u) \wedge \mu_K(s,u))/(u \wedge v),$$

and

$$\nu_{J\cap K}(s,w) = \int_{u \in i_s^u} \int_{v \in i_s^v} (\nu_J(s,u) \vee \nu_K(s,u))/(u \wedge v),$$

Example 7. *Let J and K be two T2IFSs representing the set “Young”. The “Youthness” is P-MF of J and K. Then the degree of “Youthness” and “Adultness” are the S-MF and S-NMF respectively. We consider both J and K to be defined on S={7,9,13} which are eventyualy represented as:*

J=((0.8,0.1)/0.7 +(0.6,0.2)/0.8 + (0.5,0.4)/0.9) /7 + ((0.7,0.2)/0.5 + (0.6,0.3) /0.6 +(0.5,0.4)/0.7) /9 +((0.8,0.2)/0.3 +(0.7,0.3)/0.4 +(0.4,0.5)/0. 5)/13

K= ((0.7,0.2)/0.6 +(0.5,0.4)/0.7 +(0.5,0.5)/0.8)/7 + ((0.8,0.2)/0.4 +(0.8,0.1)/0.5 +(0.4,0.5)/0.6)/9 +((0.7,0.3)/0.2 +(0.6,0.3)/0.3 +(0.4,0.4)/0.4)13.

Now for 7, S-M and S-NM of J and K are
((0.8,0.1)/0.7 +(0.6,0.2)/0.8 +(0.5,0.4)/0.9)/7
and
((0.7,0.2)/0.6 +(0.5,0.4)/0.7 +(0.5,0.5)/0.8)/7
for S=7, the union of J and K is $(\mu_{J\cup K}(7), \nu_{J\cup K}(7))$
((0.8,0.1)/0.7 +(0.6,0.2)/0.8 +(0.5,0.4)/0.9)/7 $\vee$ *((0.7,0.2)/0.6 +(0.5,0.4)/0.7 + (0.5,0.5)/0.8)/7* $((0.8 \wedge 0.7), (0.1 \vee 0.2))/(0.7 \vee 0.6)$ $+((0.8 \wedge 0.5), (0.1 \vee 0.4))/(0.7 \vee 0.7)$ $+((0.8 \wedge 0.5), (0.1 \vee 0.5))/(0.7 \vee 0.8)+$ $+((0.6 \wedge 0.7), (0.2 \vee 0.2))/(0.8 \vee 0.6)$ $+((0.6 \wedge 0.5), (0.2 \vee 0.4))/(0.8 \vee 0.7)$ $+((0.6 \wedge 0.5), (0.2 \vee 0.5))/(0.8 \vee 0.8)$ $+((0.5 \wedge 0.7), (0.4 \vee 0.2))/(0.9 \vee 0.6)$ $+((0.5 \wedge 0.5), (0.4 \vee 0.4))/(0.9 \vee 0.7)$ $+((0.5 \wedge 0.5), (0.4 \vee 0.5))/(0.9 \vee 0.8)$
=(0.5,0.4)/0.7 + (0.5,0.5)/0.8 + (0.6,0.2)/0.8 +(0.5,0.4)/0.8 +(0.5,0.5)/0.8 +(0.5,0.4)/0.9 +(0.5,0.4)/0.9 + (0.5,0.5)/0.9
*= (0.5,0.4)/0.7+(*max*(0.5,0.6,0.5,0.5),* min*(0.5,0.2,0.4,0.5)) 0.8 +* (max*(0.5,0.5,0.5),*min*(0.4,0.4,0.5))/0.9*
=(0.5,0.4)/0.7 + (0.6,0.2)/0.8 + (0.5,0.4)/0.9

Analysis on operations of union and intersection for different fuzzy sets

Fuzzy sets use a membership function to assign a degree of membership to each element of a set. This allows for a more flexible and nuanced representation of uncertainty than the binary membership characteristic of classical sets. The union and intersection operations of fuzzy sets are defined by taking the maximum and minimum of the membership functions, respectively.

Type-2 fuzzy sets take this idea one step further, by allowing the membership function itself to be a fuzzy set. This enables an even more sophisticated representation of uncertainty, but also makes the union and intersection operations more complex.

Intuitionistic fuzzy sets go beyond the binary membership characteristic of fuzzy sets and also incorporate a degree of non-membership. This allows for a more nuanced representation of uncertainty, particularly when dealing with vague or ambiguous information. The union and intersection operations of intuitionistic fuzzy sets take into account both membership and non-membership degrees.

Type-2 intuitionistic fuzzy sets combine the concepts of T2FS and IFS, allowing for an even more sophisticated representation ofOverall, these set types offer a rich and powerful toolbox for dealing with uncertainty and imprecision in a wide range of applications, including decision making, data analysis, and control systems. uncertainty. The union and intersection operations of T2IFSs also take into account both membership and non-membership degrees, making them particularly useful for handling uncertain or ambiguous information.

Results of Comparison

As we compared different fuzzy sets on the basis of union and intersection every fuzzy set has their importance, but we found that type-2 intuitionistic fuzzy sets offer a best tool for solving decision making problems. In terms of operations, type-2 intuitionistic fuzzy sets exhibit differences compared to other fuzzy sets. The union and intersection operations for type-2 intuitionistic fuzzy sets involve considering the lower and upper membership and non-membership values separately. This allows for a more flexible and granular manipulation of fuzzy sets, enabling decision-makers to capture the various degrees of uncertainty and ambiguity inherent in complex decision problems.

3.6. Comparison on the basis of distance measures

Distance measure between FSs and T2FSs

Definition 20. *Distance measure plays an important role in decision making. Let* $F_1(S)$ *be the class of all T1FS of S.* $\mu_J(s) \rightarrow [0,1]$*is the M-F of S in* $F_1(S)$*. Let's consider two FSs J and K in* $F_1(S)$*. Then d(J,K) is said to be a distance measure between J and K if*

$$d\colon\ F_1(S) \times F_1(S) \rightarrow [0,1] \tag{29}$$

satisfies following axioms.

$$(p1) \quad 0 \leq d(J,K) \leq 1 \; \forall \; J,K \in F_1(S) \tag{30}$$

$$(p2) \quad d(J,K) = d(K,J) \tag{31}$$

$$(p3) \quad d(J,K) = 0 \quad if \;\; J = K \tag{32}$$

$$(p4) \quad d(J,K) = 0, d(J,L) = 0, L \in F_1(S) \;\; then \;\; d(K,L) = 0. \tag{33}$$

For two FSs J and K, the following distance measure is provided. [14]
Hamming distance

$$d_{1h}(J,K) = \frac{1}{n}\sum_{j=1}^{n} |\mu_J(s_j) - \mu_K(s_j)| \tag{34}$$

Euclidian distance

$$d_{1e}(J,K) = \{\frac{1}{n}\sum_{j=1}^{n} |\mu_J(s_j) - \mu_K(s_j)|^2\}^{1/2} \tag{35}$$

3.7. Numerical Example

Lets consider four kinds of metal fields and each field is featured by five metals . We can express these four fields by FSs $\{c_1, c_2, c_3, c_4\}$ in space $\{S = s_1, s_2, s_3, s_4, s_5\}$.See Table 3. There is another kind of special metal $\{n\}$ so we have to find which metal field this metal belongs.

Table 3

	s_1	s_2	s_3	s_4	s_5
$u_{c_1}(s)$	1	0.7	0.5	0.7	1
$u_{c_2}(s)$	1.0	0.7	0.9	0.9	0.9
$u_{c_3}(s)$	1.0	0.9	1.0	0.9	0.9
$u_{c_4}(s)$	0.9	0.9	0.9	0.2	0.7
$u_n(s)$	0.9	0.2	0.2	0.2	0.9

we have

$$d_{1h}(J, K) = \frac{1}{5} \sum_{j=1}^{5} |\mu_J(s_j) - \mu_K(s_j)| \tag{36}$$

since from the Table 3 and using $d_{1h}(J, K)$ we get following result

$d_{1h}(c_1, n) = 0.3, d_{1h}(c_2, n) = 0.4, d_{1h}(c_3, n) = 0.575, d_{1h}(c_4, n) = 0.32$

which implies special metal n is produced from metal field c_1

for T1FS we have only M-F but for T2FS we have P-MF,S-MF and FOU.

[29] Examine the following factors in order to calculate the distance measure for T2FSs. P-MF, S-MF and FOU in the currently used distance measure the following distance measure is defined for type-2 fuzzy sets J and K.

$$d_{2h}(J, K) = \frac{1}{2n} \sum_{j=1}^{n} |u_J(s_j) - u_K(s_j)| + |f_{sj}(u_J) - f_{sj}(u_k)| + |\xi_J(s_j) - \xi_K(s_j)| \tag{37}$$

3.8. Numerical Example

Let's consider four kinds of metal fields and each field is featured by five metals . We can express these four fields by T2FSs $\{c_1, c_2, c_3, c_4\}$ in space $\{S = s_1, s_2, s_3, s_4, s_5\}$.See Table 3. There is another kind of special metal $\{n\}$ so we have to find which metal field this metal belongs.

Table 4

	s_1	s_2	s_3	s_4	s_5
$u_{c_1}(s)$	1	0.7	0.5	0.7	1
$f_s(u_{c_1})$	0.7	0.9	0.2	0.5	0.9
$u_{c_2}(s)$	1.0	0.7	0.9	0.9	0.9
$f_s(u_{c_2})$	0.9	0.7	1.0	0.7	0.7
$u_{c_3}(s)$	1.0	0.9	1.0	0.9	0.9
$f_s(u_{c_3})$	0.7	1.0	0.9	0.9	0.4
$u_{c_4}(s)$	0.9	0.9	0.9	0.2	0.7
$f_s(u_{c_4})$	1.0	0.7	0.5	0.0	0.4
$u_n(s)$	0.9	0.2	0.2	0.2	0.9
$f_s(u_n)$	0.4	0.5	0.4	0.0	0.7

we have

$$d_{2h}(J,K) = \frac{1}{2n}\sum_{j=1}^{n} |u_J(s_j) - u_K(s_j)| + |f_{sj}(u_J) - f_{sj}(u_k)| + |\xi_J(s_j) - \xi_K(s_j)| \tag{38}$$

since from the Table 4 and using $d_{2h}(J,K)$ we get following result
$d_{2h}(c_1,n) = 0.44, d_{2h}(c_2,n) = 0.48, d_{2h}(c_3,n) = 0.6, d_{2h}(c_4,n) = 0.46$
which implies special metal n is produced from metal field c_1.

Distance measures between IFS

Definition 21. *Some new distance measures between IFSs has been defined By [39] Let J and K be two IFS in* $S = \{s_1, s_2, ...s_n\}$

$$d_3(J,K) = \frac{1}{n}\sum_{i=1}^{n} \frac{|\mu_J(s_i) - \mu_K(s_i)| + |\nu_J(s_i) - \nu_K(s_i)|}{4} + \frac{max(|\mu_J(s_i) - \mu_K(s_i)|, |\nu_J(s_i) - \nu_K(s_i)|)}{2} \tag{39}$$

where $J = \{s_i, \mu_J(s_i), \nu_J(s_i)|s_i \in S\}, K = \{s_i, \mu_K(s_i), \nu_K(s_i)|s_i \in S\}$

3.9. Numerical Example

Lets consider four kinds of metal fields and each field is featured by five metals . We can express these four fields by T2IFSs $\{c_1, c_2, c_3, c_4\}$ in space $\{S = s_1, s_2, s_3, s_4, s_5\}$.See Table 5. There is another kind of special metal $\{n\}$ so we have to find which metal field this metal belongs.

Table 5

	x_1	x_2	x_3	x_4	x_5
$u_{c_1}(x)$	1	0.7	0.5	0.7	1
$v_{c_1}(x)$	0	0.1	0.4	0.2	0
$u_{c_2}(x)$	1.0	0.7	0.9	0.9	0.9
$v_{c_2}(x)$	0	0.4	0.1	0.1	0.1
$u_{c_3}(x)$	1.0	0.9	1.0	0.9	0.9
$v_{c_3}(x)$	0.0	0.1	0.0	0.1	0.1
$u_{c_4}(x)$	0.9	0.9	0.9	0.2	0.7
$v_{c_4}(x)$	0.1	0.0	0.1	0.7	0.2
$u_n(x)$	0.9	0.2	0.2	0.2	0.9
$v_n(x)$	0.1	0.7	0.7	0.7	0.0

we have

$$d_3(J,K) = \frac{1}{n}\sum_{i=1}^{n} \frac{|\mu_J(s_i)-\mu_K(s_i)|+|\nu_J(s_i)-\nu_K(s_i)|}{4} + \frac{max(|\mu_J(s_i)-\mu_K(s_i)|,|\nu_J(s_i)-\nu_K(s_i)|)}{2} \quad (40)$$

since from the Table 4 and using $d_2(P,Q)$ we get following result

$d_3(c_1,n) = 0.305, d_3(c_2,n) = 0.285, d_3(c_3,n) = 0.460, d_3(c_4,n) = 0.315$

which implies special metal n is produced from metal field c_2.

Definition 22. *[28] The variance margin function (V-MF) of T2IFS is defined as the difference between P-MF and S-MF, P-NMF and S-NMF. It is denoted by η and ξ respectively.*

Now we extended this new distance measure for T2IFSs and provided the comparison between this distance measure with exixting distance measure with a numerical example.

4. New Distance measures between T2IFS

Firstly we analyse the definition of "distance measure for T2IFS". Singh, S., & Garg, H. [28] defined the concept for T2IFS where they used triangle inequality and we defined the inclusion relation between T2IFS which is not satisfied by euclidean distance measure It is necessary to establish the inclusion relation between T2IFS, so we introduced a new distance measure which satisfies inclusion relation in T2IFS.

For convenience, two T2IFSs P and Q in T are denoted by $P = \{t(u, f_{tj}(u_P), (v, g_{tj}(v_P))|t \in T\}$ and $Q = \{t(u, f_{tj}(u_Q), (v, g_{tj}(v_Q))|t \in T\}$ then we defined new distance for P and Q by considering the P-MF,S-MF,P-NMF and S-NMF

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$$d_4(P,Q)=\frac{1}{2n}\sum_{i=1}^{n}\frac{|u_P(t_i)-u_Q(t_i)|+|v_P(t_i)-v_Q(t_i)|+|f_{ti}(u_P)-f_{ti}(u_Q)|+|g_{ti}(u_P)-g_{ti}(u_Q)|}{4} + \frac{max|u_P(t_i)-u_Q(t_i)|,|v_P(t_i)-v_Q(t_i)|,|f_{ti}(u_P)-f_{ti}(u_Q)|,|g_{ti}(u_P)-g_{ti}(u_Q)|}{2} \tag{41}$$

Definition 23. *A real function $d_4\colon F_2^I(t)\times F_2^I(t)\to[0,1]$ is called distance measure, where d_4 satisfies the following axioms:*

$$(p1)\quad 0\le d_4(P,Q)\le 1,\ \forall\ (P,Q)\in F_2^I(t) \tag{42}$$

$$(p2)\quad d_4(P,Q)=0,\ IF\quad P=Q \tag{43}$$

$$(p3)\quad d_4(P,Q)=d_4(Q,P) \tag{44}$$

$$(p4)\quad P\subseteq Q\subseteq R\ \ where\ \ P,Q,R\in F_2^I(t),\ \ then\ \ d_4(P,R)\ge d_4(P,Q)\ \ and\ \ d_4(P,R)\ge d_4(Q,R). \tag{45}$$

Now we will prove the above defined measure is a valid distance measure for T2IFS. condition (P_1)given in eq 27

$$(P_1)\implies 0\le d_4(P,Q)\le 1$$

Let P and Q be two T2IFS then we have
$|u_P(t_i)-u_Q(t_i)|\ge 0, |f_{ti}(u_P)-f_{ti}(u_Q)|\ge 0$

$|v_P(t_i)-v_Q(t_i)|\ge 0, |g_{ti}(u_P)-g_{ti}(u_Q)|\ge 0$
this implies $d_2(P,Q)\ge 0$
then we have $|u_P(t_i)-u_Q(t_i)|\le 1, |f_{ti}(u_P)-f_{ti}(u_Q)|\le 1$

$|v_P(t_i)-v_Q(t_i)|\le 1, |g_{ti}(u_P)-g_{ti}(u_Q)|\le 1$
$\implies d_4(P,Q)\le 1$ hence

$$0\le d_4(P,Q)\le 1$$

condition (P_2) given by eq 28 follows trivialy so we prove for (P_3) and (P_4) condition given in eq 29 and 30 respectively.

$$(P_3)\implies d_4(P,Q)=d_4(Q,P)$$

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we have

$$d_4(P,Q) = \frac{1}{2n}\sum_{i=1}^{n}\frac{|u_P(t_i)-u_Q(t_i)|+|v_P(t_i)-v_Q(t_i)|+|f_{ti}(u_P)-f_{ti}(u_Q)|+|g_{ti}(u_P)-g_{ti}(u_Q)|}{4} + \frac{max|u_P(t_i)-u_Q(t_i)|,|v_P(t_i)-v_Q(t_i)|,|f_{ti}(u_P)-f_{ti}(u_Q)|,|g_{ti}(u_P)-g_{ti}(u_Q)|}{2} \tag{46}$$

$$= \frac{1}{2n}\sum_{i=1}^{n}\frac{|u_Q(t_i)-u_P(t_i)|+|v_Q(t_i)-v_P(t_i)|+|f_{ti}(u_Q)-f_{ti}(u_P)|+|g_{ti}(u_Q)-g_{ti}(u_P)|}{4} + \frac{max|u_Q(t_i)-u_P(t_i)|,|v_Q(t_i)-v_P(t_i)|,|f_{ti}(u_Q)-f_{ti}(u_P)|,|g_{ti}(u_Q)-g_{ti}(u_P)|}{2} \tag{47}$$

$$= d_4(Q,P)$$

$$\implies d_4(P,Q) = d_4(Q,P)$$

Now to prove (P_4)

$$(P_4) \implies d_4(P,R) \geq d_4(P,Q) \tag{48}$$

it is easy to see that $|u_P(t_i)-u_R(t_i)| \geq |u_P(t_i)-u_Q(t_i)|, |f_{ti}(u_P)-f_{ti}(u_R)| \geq |f_{ti}(u_P)-f_{ti}(u_Q)|$ $|v_P(t_i)-v_R(t_i)| \geq |v_P(t_i)-v_Q(t_i)|, |g_{ti}(u_P)-g_{ti}(u_R)| \geq |g_{ti}(u_P)-g_{ti}(u_Q)|$ so we have

$$\frac{1}{2n}\sum_{i=1}^{n}\frac{|u_P(t_i)-u_R(t_i)|+|v_P(t_i)-v_R(t_i)|+|f_{ti}(u_P)-f_{ti}(u_R)|+|g_{ti}(u_P)-g_{ti}(u_R)|}{4} + \frac{max|u_P(t_i)-u_R(t_i)|,|v_P(t_i)-v_R(t_i)|,|f_{ti}(u_P)-f_{ti}(u_R)|,|g_{ti}(u_P)-g_{ti}(u_R)|}{2} \tag{49}$$

$$\geq \frac{1}{2n}\sum_{i=1}^{n}\frac{|u_P(t_i)-u_Q(t_i)|+|v_P(t_i)-v_Q(t_i)|+|f_{ti}(u_P)-f_{ti}(u_Q)|+|g_{ti}(u_P)-g_{ti}(u_Q)|}{4} + \frac{max|u_P(t_i)-u_Q(t_i)|,|v_P(t_i)-v_Q(t_i)|,|f_{ti}(u_P)-f_{ti}(u_Q)|,|g_{ti}(u_P)-g_{ti}(u_Q)|}{2} \tag{50}$$

then we get inequality $d_4(P,R) \geq d_4(P,Q)$ similarly we can prove $d_4(P,R) \geq d_4(Q,R)$ hence satisfies condition (P_4) so we proved this is a valid distance measure for T2IFS

4.1. Numerical Example

Lets consider four kinds of metal fields and each field is featured by five metals . We can express these four fields by T2IFSs $\{c_1, c_2, c_3, c_4\}$ in space $\{T = t_1, t_2, t_3, t_4, t_5\}$.See Table 6. There is another kind of special metal $\{n\}$ so we have to find which metal field this metal belongs.

Table 6

	t_1	t_2	t_3	t_4	t_5
$u_{c_1}(t)$	1	0.7	0.5	0.7	1
$f_t(u_{c_1})$	0.7	0.9	0.2	0.5	0.9
$v_{c_1}(t)$	0	0.1	0.4	0.2	0
$g_t(u_{c_1})$	0.2	0.1	0.5	0.4	0.1
$u_{c_2}(t)$	1.0	0.7	0.9	0.9	0.9
$f_t(u_{c_2})$	0.9	0.7	1.0	0.7	0.7
$v_{c_2}(t)$	0	0.4	0.1	0.1	0.1
$g_t(u_{c_2})$	0.1	0.4	0	0.2	0.2
$u_{c_3}(t)$	1.0	0.9	1.0	0.9	0.9
$f_t(u_{c_3})$	0.7	1.0	0.9	0.9	0.4
$v_{c_3}(t)$	0.0	0.1	0.0	0.1	0.1
$g_t(u_{c_3})$	0.2	0	0.1	0.1	0.5
$u_{c_4}(t)$	0.9	0.9	0.9	0.2	0.7
$f_t(u_{c_4})$	1.0	0.7	0.5	0.0	0.4
$v_{c_4}(t)$	0.1	0.0	0.1	0.7	0.2
$g_t(u_{c_4})$	0	0.1	0.4	1.0	0.5
$u_n(t)$	0.9	0.2	0.2	0.2	0.9
$f_t(u_n)$	0.4	0.5	0.4	0.0	0.7
$v_n(t)$	0.1	0.7	0.7	0.7	0.0
$g_t(u_n)$	0.5	0.4	0.5	1.0	0.1

we have

$$d_4(P,Q) = \frac{1}{2n}\sum_{i=1}^{n} \frac{|u_P(t_i) - u_Q(t_i)| + |v_P(t_i) - v_Q(t_i)| + |f_{ti}(u_P) - f_{ti}(u_Q)| + |g_{ti}(u_P) - g_{ti}(u_Q)|}{4} + \frac{max|u_P(t_i) - u_Q(t_i)|, |v_P(t_i) - v_Q(t_i)|, |f_{ti}(u_P) - f_{ti}(u_Q)|, |g_{ti}(u_P) - g_{ti}(u_Q)|}{2} \tag{51}$$

since from the Table 4 and using $d_2(P,Q)$ we get following result

$d_2(c_1,n) = 0.275, d_2(c_2,n) = 0.312, d_2(c_3,n) = 0.385, d_2(c_4,n) = 0.259$

which implies special metal n is produced from metal field c_4 obviously this coincides with the result of Sukhveer Singh and Harish Garg [28] but there approach is not valid for some calculations as it gives value beyond 1.0 which means our approach is better and also our approach includes inclusion relation which is stronger than triangle inequality.

Analysis on the basis of distance measure for different fuzzy sets

Type-1 fuzzy sets (T1FSs) are distinguished by membership functions that are created using the degree of membership between each element, set in the range [0, 1]. Yet, a wide variety of recent publications on decision-making issues have taken intuitionistic fuzzy sets (IFSs) into account to handle the ambiguity. IFSs are the generalised version of fuzzy sets proposed by Atanassov [2], which gives the freedom to also model the reluctance in the decision-making). They are specified by a membership and a non-membership degree, and the hesitation margin is obtained by subtracting both from unity. Yet, as these traditional T1FSs or IFSs still have crisp membership values, they are frequently linked to interpretability problems. There is a membership and a non-membership in type-1 when dealing with these classical intuitionistic fuzzy sets, and it is thought that the uncertainty in the evaluation can be seen of as dissipating. There may still be some confusion close to the membership and non-membership boundaries, though. Moreover, confusing and imprecise information tends to be more prevalent in real-world application contexts. Type-2 membership function can be used to solve this issue, as type-2 fuzzy sets demonstrate (T2FSs). It can be easily seen from the above defined two examples for T1IFS and T2IFS respectively. In first example we use only membership and non-membership values but in 2nd example we take secondary membership and secondary non-membership values into consideration, so it better to use T2IFS instead of T1IFS when the uncertainty is so high. We analysed different fuzzy sets and calculated the distance measures between these sets by using numerical examples to check out the comparison and we found that T2IFS are better.

Results of Comparison

To understand their importance, a comparison based on distance measures was conducted, using examples for each type of fuzzy set. Distance measures provide a quantitative assessment of similarity or dissimilarity between fuzzy sets. Through these examples, it becomes apparent that T2IFSs outperform the other fuzzy sets when faced with ambiguous or uncertain information.

5. Conclusion

Operation of union and intersection between T1FS,T2FS,IFS and T2IFS is discussed with the help of examples, to understand the importance of these fuzzy sets a comparison is made on the basis of distance measures by the aid of examples on each above defined fuzzy sets. However, it is worth noting that the existing distance measures for T2IFSs have limitations. To address this, a new distance measure is proposed specifically tailored for T2IFSs. This measure overcomes the limitations of the existing one, enabling a more accurate and reliable comparison of T2IFSs. In conclusion, when faced with decision-making scenarios where information is ambiguous or uncertain, it is better to utilize T2IFSs. Their ability to consider both membership and non-membership values, along with the proposed improved distance measure, allows for a more comprehensive and effective analysis of fuzzy information. By employing T2IFSs in such conditions, decision-makers can obtain more reliable and informed outcomes, leading to better decision-making overall.

Conflict of interest

The authors declares no conflict of interest.

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